CERTAIN INVESTIGATIONS IN ALTERNATIVE THEORIES OF GRAVITATION

SYNOPSIS

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SYNOPSIS

General relativity or Einstein's theory of gravitation is a corner stone of modern physics and is of major importance in its application to cosmology, which deals with the large scale structure of the universe. General theory of relativity is a geometric theory of gravitation proposed by Albert Einstein (1916). It is the correct description of gravitation in modern physics. General relativity generalizes special relativity and Newton's law of universal gravitation, providing a unified description of gravity as a geometric property of space and time. In particular, the curvature of space time is directly related to the four-momentum of matter and radiation. The relation is specified by the Einstein field equations, a system of partial differential equations. Einstein's theory of gravitation is based on Riemannian metric tensor g_{ij} which describes not only the gravitational field but also the geometry. This theory of gravitation has great formal beauty and mathematical elegance and is found to lead to a complete theory of gravitation.

Many predictions of general relativity differ significantly from those of classical physics, especially concerning the passage of time, the geometry of space, the motion of bodies in free fall, and the propagation of light. Examples of such differences include gravitational time dilation, the gravitational redshift of light, and the gravitational time delay. General relativity predictions have been

confirmed observationally and experimentally. Although general relativity is not the only relativistic theory of gravity, it is the simplest theory that is consistent with experimental data.

Since Einstein published his theory of gravitation there have been many criticisms of general relativity because of the lack of certain desirable features in this theory. For example, Einstein himself pointed out that general relativity does not account satisfactorily for the inertial properties of matter, i.e. Mach's principle is not substantiated by general relativity. So in recent years, there have been some interesting attempts to generalize the general theory of relativity by incorporating Mach's principle and other certain desired features which are lacking in the original theory. With this objective, various versions of scalar-tensor theories of gravitational fields have been suggested and widely discussed. Among these theories which are attracting more and more attention from physicists are the theories proposed by Brans and Dicke (1961), Nordtvedt (1970), Saez and Ballester (1986) and Barber (1982). Extensive examination of certain aspects of the Scalar-tensor theories of gravitation (e.g. exact solutions for field equations, cosmology, the three crucial tests of general relativity etc.) has revealed significant points of similarity and contrast between Scalar-tensor theories and Einstein's theory of gravitation. All versions of Scalar-tensor theories are based on the introduction of a scalar field variable into the formulation of general relativity and this

scalar field together with the metric tensor field g_{ij} forms the scalar-tensor field, representing the gravitational field.

Brans and Dicke (1961) introduced a scalar-tensor theory of gravitation involving a scalar function in addition to the familiar general relativistic metric tensor. In this theory the scalar field has the dimension of inverse of the gravitational constant and its role is confined to its effects on gravitational field equations.

Barber (1982) has proposed two theories in (1982). The first one is a modified Brans and Dicke theory that is unsatisfactory since the equivalence principle is violated. The second one is an adaptation of general relativity to include continuous creation and is within the limits of the observations. These modified theories create the universe out of self contained gravitational and matter fields. Brans (1987) has pointed out that Barber's first theory is not only in disagreement with experiment, but is actually inconsistent. In this theory the scalar field does not directly gravitate, but simply divides the matter tensor, acting as a reciprocal gravitational constant. It is postulated that the scalar field couples to the trace of the energy momentum tensor. Anisotropic homogeneous universes play an important role in understanding some essential features of the universe, such as the formation of galaxies during its early stages of evolution.

Subsequently Saez and Ballester (1986) developed a scalar-tensor theory in which the metric is coupled with a dimension less

scalar field. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory suggests a positive way to solve the missing matter problem in non-flat FRW cosmologies.

In recent years there has been a lot of interest in alternative theories of gravitation. Noteworthy among them is the theory of gravitation proposed by Sen (1957) based on Lyra (1951), using a modified Riemannian geometry in which a gauge function has been introduced into the structure less manifold as a result of which the cosmological constant arises naturally from the geometry. In general relativity, Einstein succeeded in geometrizing gravitation by identifying the metric tensor with gravitational potentials. In scalar tensor theory of Brans – Dicke on the other hand, the scalar field remains alien to the geometry. Lyra's geometry is more in keeping with spirit of Einstein's principle of geometrisation since both the scalar and tensor fields have more or less intrinsic geometrical significance.

With the advent of scalar tensor theories of gravitation, in recent years there has been a lot of interest in the study of the various aspects of these scalar-tensor theories and then to compare them with the results of general theory of relativity. With this motivation we have taken up the study of certain investigations of physical interest in alternative theories of gravitation.

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic in which a process of

isotropization of universe is studied through the passage of time. Moreover from the theoretical point of view anisotropic universes have a greater generality than isotropic models. The simplicity of the field equations and relative ease of solutions made Bianchi space times useful in constructing models of spatially homogeneous and anisotropic cosmologies.

This thesis entitled **"CERTAIN INVESTIGATIONS IN ALTERNATIVE THEORIES OF GRAVITATION"** comprises of eight chapters and is devoted to the study of some cosmological models of physical interest in alternative theories of gravitation.

Chapter–I is introductory in nature in which a systematic survey of scalar tensor theories of gravitation, formulated by Brans – Dicke (1961), Barber (1982), Saez - Ballester (1986) and Lyra (1951) has been conducted which are investigated in the thesis. Subsequently a brief review of Bianchi space times is presented. These are relevant and necessary to understand the work contained in the subsequent chapters. Finally a brief discussion on the problems investigated is presented.

In Chapter-II, at the outset, a discussion of spatially homogeneous Bianchi type-II, VIII & IX space times in the presence of a bulk viscous fluid containing one dimensional cosmic strings in the frame work of a scalar-tensor theory proposed by Brans and Dicke (1961). Subsequently the field equations in string cosmological models with bulk viscosity are solved. With the solutions obtained Bianchi

type-II, VIII & IX bulk viscous string cosmological models in Brans-Dicke theory of gravitation are presented. Some important features of the models are also discussed.

Chapter-III consists of discussion on the axially symmetric perfect fluid as well as strange quark matter attached to string cloud cosmological models in Brans-Dicke theory of gravitation and string cosmological model with bulk viscosity in self creation theory of gravitation. Field equations are solved in each of the above cases. Exact solutions of field equations are obtained and presented. Some important features of the models are also discussed.

Chapter-IV is devoted to the study of higher dimensional string cosmological models in a scalar tensor theory of gravitation proposed by Brans and Dicke (1961). Subsequently the field equations have been obtained using a five dimensional Kaluza-Klein metric. A new class of solutions of the field equations is presented. Some important features of the models are also discussed.

Chapter-V is mainly concerned with the discussion of a spatially homogeneous Bianchi type-II, VIII and IX cosmological models with strange quark matter attached to string cloud in Brans-Dicke and general theory of gravitation. Field equations are obtained in the presence of quark matter. Some important features of the models are also discussed.

Chapter-VI consists of a discussion of Bianchi type- II, VIII & IX magnetized cosmological models in a scalar tensor theory of gravitation based on Lyra geometry. Here the magnetic field is due to an electric current along the z-axis. The distribution consists of an infinite electrical conductivity in the presence of a magnetic field. Some important features of the models are also discussed.

Chapter-VII deals with Bianchi type- II, VIII & IX space times in string cosmological models with bulk viscosity in a scalar tensor theory of gravitation based on Lyra geometry. Subsequently the field equations have been obtained using Bianchi type- II, VIII & IX metrics. A new class of solutions of the field equations is presented. Some important features of the models are also discussed.

Chapter-VIII is devoted to the study of a spatially homogeneous anisotropic Bianchi type - II, VIII & IX as well as isotropic space times filled with perfect fluid and dark energy possessing dynamical energy density in Saez - Ballester (1986) scalar tensor theory of gravitation and general relativity. Field equations are obtained in the presence of perfect fluid and dark energy for the Bianchi type- II, VIII & IX metrics. Exact solutions of the field equations are presented. Some physical and geometrical properties of the models are also discussed.

Spatially homogeneous and anisotropic exact cosmological models of Bianchi type are, in general, useful to the study of evolution of the universe. The above exact cosmological models in certain scalar-

tensor theories of gravitation will help, largely, to understand cosmology at the early stages of evolution of the universe. The work presented in this thesis will be also useful to have a comparative study of the cosmological models of Einstein's theory of gravitation and alternative theories of gravitation.

LIST OF PUBLICATIONS

1.* An exact Bianchi type-V perfect fluid cosmological model in Brans Dicke theory of gravitation: Astrophysics and Space Sci. **315**, 211 (2008).

2.* Bianchi type-I string cosmological models in Bimetric theory of gravitation: Astrophysics and Space Sci. **317**, 79 (2008).

3.* Axially symmetric string cosmological models in Brans-Dicke theory of Gravitation: Astrophysics and Space Sci. 323, 401 (2009).

4.* Anisotropic universe with cosmic strings and bulk viscosity in Saez- Ballester scalar-tensor theory of gravitation: Astrophysics and Space Sci., **335**, 635 (2011).

5. Bianchi Types II, VIII, and IX string cosmological models with bulk viscosity in a theory of gravitation: ISRN Mathematical Physics,DOI:10.5402/2012/341612 (2012).

6. A higher-dimensional string cosmological model in a scalar-tensor theory of gravitation: Eur. Phys. J. Plus, **127**, 33 (2012).

7. Axially symmetric string cosmological model with bulk viscosity in Self creation theory of gravitation: Eur. Phys. J. Plus, **127**, 49 (2012).

8. Bianchi Types II, VIII & IX string cosmological models with bulk viscosity in Brans-Dicke theory of gravitation: International Journal of Theoretical Physics, **51**, 3013 (2012).

9. Axially Symmetric perfect fluid cosmological models in Brans-Dicke scalar tensor theory of gravitation: The African Review of Physics, **7**:0042 (2012).

10. Axially symmetric space-time with strange quark matter attached to string cloud in Brans–Dicke theory of gravitation: International Journal of Theoretical physics, DOI 10.1007/s10773-012-1420-3 (2012).

11. Bianchi Type-II, VIII & IX cosmological models with strange quark matter attached to string cloud in Brans–Dicke and General theory of gravitation: International Journal of Theoretical Physics, DOI 10.1007/s10773-012-1439-5 (2012).

12.* Bianchi type - II, VIII & IX dark energy cosmological models in Saez - Ballester theory of gravitation: The African Review of Physics (accepted for publication) (2012).

13. Bianchi Type-II, VIII & IX magnetized cosmological models in Lyra Geometry: Prespacetime journal, **3**, 15 (2012).

14. Bianchi type - II, VIII & IX perfect fluid dark energy cosmological models in Saez - Ballester and General theory of gravitation: ISRN Astronomy and Astrophysics, DOI 10.1155/2013/924834 (2013).

* These papers are not included in this thesis.